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JEE 2017 through Python

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***Abstract*—This manual is a collection of math problems from the JEE 2017 mains paper, suitably modified as programming problems. These problems can be solved using the Python scripts in the JEE 2016 manual. This will give the student enough practice in Python programming.**

Using the python codes in the JEE 2016 manual, solve the following problems.

Problem 1. Find the sum of all the real values of x satisfying the equation

$$2^{(x-1)\left(x^2+5x-50\right)} = 1 \tag{1.1}$$

Problem 2. Sketch the curve

$$\operatorname{Im}\left(\frac{\mathrm{J}z-2}{z-1}\right)+1=0, z \in \mathbf{C}, z \neq 1 \tag{2.1}$$

Problem 3. If $x = a, y = b, z = c$ is a solution of the system of linear equations

$$\begin{aligned} x+8y+7z &= 0 \\ 9x+2y+3z &= 0 \\ x+y+z &= 0 \end{aligned} \tag{3.1}$$

such that the point (a, b, c) lies on the plane $x+2y+z = 6$, then find $2a + b + c$.

Problem 4. a, b, c are in AP such that $abc = 8$. Find the minimum value of b.

Problem 5. Given

$$S_n = \frac{1}{1^3} + \frac{1+2}{1^3+2^3} + \frac{1+2+3}{1^3+2^3+3^3} + \cdots + \frac{1+2+\cdots+n}{1^3+2^3+\cdots+3^3} \tag{5.1}$$

and $100S_n = n$, find n.

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Problem 6. Find the value of k for which the function

$$f(x) = \begin{cases} \left(\frac{4}{5}\right)^{\frac{\tan 4x}{\tan 5x}}, & 0 < x < \frac{\pi}{2} \\ k + \frac{2}{5}, & x = \frac{\pi}{2} \end{cases} \tag{6.1}$$

is continuous at $x = \frac{\pi}{2}$.

Problem 7. Sketch the function defined by

$$f(x) = x^3 - 3x^2 + 5x + 7 \tag{7.1}$$

and mark the regions where it is increasing or decreasing.

Problem 8. Given that

$$\lim_{n\to\infty} \frac{1^a + 2^a + \cdots + n^a}{(n+1)^{a-1}\left[(na+1)+(na+2)+\cdots+(na+n)\right]} = \frac{1}{60} \tag{8.1}$$

find a.

Problem 9. A square of each side 2, lies above the x-axis and has one vertex at the origin. If one of the sides passing through the origin makes an angle 30° with the positive direction of the x-axis, then find the sum of the x-coordinates of the vertices of the square.

Problem 10. A line drawn through the point $P(4, 7)$ cuts the circle $x^2 + y^2 = 9$ at the points A and B. Find $PA.PB$.

Problem 11. Find the eccentricity of an ellipse having centre at the origin, axes along the coordinate axes and passing through the points $(4, -1)$ and $(-2, 2)$

Problem 12. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then find $|c|$.

Problem 13. Solve

$$\sin\left[\cot^{-1}(1+x)\right] = \cos\left[\tan^{-1}x\right] \tag{13.1}$$

Problem 14. Sketch the curve

$$2\left|z+3\mathrm{J}\right| - \left|z-\mathrm{J}\right| = 0, z \in \mathbf{C} \tag{14.1}$$

Problem 15. Find λ for which the system of equations

$$\begin{aligned} 2x+4y-\lambda z &= 0 \\ 4x+\lambda y+2z &= 0 \\ \lambda x+2y+2z &= 0 \end{aligned} \tag{15.1}$$

has infinitely many solutions

Problem 16. The sum of the first n terms of the series

$$\sqrt{3}+\sqrt{75}+\sqrt{243}+\sqrt{507}+\cdots = 435\sqrt{3}. \tag{16.1}$$

Find n.

Problem 17. Find

$$\lim_{x\to 3}\frac{\sqrt{3x}-3}{\sqrt{2x-4}-\sqrt{2}} \tag{17.1}$$

Problem 18. The tangent at the point $(2,-2)$ to the find k. curve, $x^2y^2-2x=4(1-y)$ does not pass through the point

1) $\left(4,\frac{1}{3}\right)$
2) $(8,5)$
3) $(-4,-9)$
4) $(-2,-7)$

Problem 19. A point P has coordinates $(0,-2)$ and Q is any point on the circle

$$x^2+y^2-5x-y+5=0 \tag{19.1}$$

Find the maximum value of PQ^2.

Problem 20. Sketch the locus of the point of intersection of the straight lines

$$\begin{aligned} tx-2y-3t &= 0 \\ x-2ty+3 &= 0, t \in \mathbf{R} \end{aligned} \tag{20.1}$$

Problem 21. A point P has co-ordinates $(0,-2)$ and Q is any point on the circle, $x^2+y^2-5x-y+5=0$, then find the maximum value of PQ^2.

Problem 22. Sketch the area of the smaller portion enclosed between the curves, $x^2+y^2=4$ and $y^2=3x$.

Problem 23. If the common tangents to the parabola, $x^2=4y$ and the circle, $x^2+y^2=4$ intersect at the point P, then find the distance of P from the origin.

Problem 24. Consider an ellipse, whose centre is at the origin and its major axis is along the x-axis. If its eccentricity is $\frac{3}{5}$ and the distance between its foci is 6, then find the area (in sq. units) of the quadrilateral inscribed in the ellipse, with the vertices as the vertices of the ellipse.

Problem 25. If

$$S = \left\{x\in[0,2\pi] : \begin{vmatrix} 0 & \cos x & -\sin x \\ \sin x & 0 & \cos x \\ \cos x & \sin x & 0 \end{vmatrix} = 0\right\} \tag{25.1}$$

then find $\sum_S \tan\left(\frac{\pi}{3}+x\right)$.

Problem 26. For a positive integer n, the quadratic equation

$$\begin{aligned} x(x+1)+(x+1)(x+2) \\ +\cdots+\left(x+\overline{n-1}\right)(x+n) = 10n \end{aligned} \tag{26.1}$$

has two consecutive integral solutions. Find n.

Problem 27. Let ω be a complex number such that $2\omega+1=z$, where $z=\sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \tag{27.1}$$

find k.

Problem 28. If

$$A = \begin{pmatrix} 2 & -3 \\ -4 & 1 \end{pmatrix}, \tag{28.1}$$

find $\operatorname{adj}\left(3A^2+12A\right)$.

Problem 29. Find the value of

$$\begin{aligned} \left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \\ \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots \\ + \left({}^{21}C_{10} - {}^{10}C_{10}\right) \end{aligned} \tag{29.1}$$

Problem 30. Find

$$\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3} \tag{30.1}$$

Problem 31. The normal to the curve $y(x-2)(x-3)=x+6$ at the point where the curve intersects the y-axis passes through the point :

1) $\left(\frac{1}{2},\frac{1}{2}\right)$

2) $\left(\frac{1}{2}, -\frac{1}{3}\right)$
3) $\left(\frac{1}{2}, \frac{1}{3}\right)$
4) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$

Problem 32. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Find the maximum area (in sq. m) of the flower-bed.

Problem 33. Sketch the region

$$\left\{(x, y) : x^2 \leq 4y, x + y \leq 3, x \geq 0, y \leq 1 + \sqrt{x}\right\}. \tag{33.1}$$

and find its area.

Problem 34. Let k be an integer such that the triangle with vertices $(k, -3k), (5, k)$ and $(-k, 2)$ has area 28 sq. units. Find the orthocentre of this triangle.

Problem 35. Find the radius of a circle, having minimum area, which touches the curve $y = 4x^2$ and the lines, $y = |x|$.

Problem 36. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = 4$, then find the equation of the normal to it at $\left(1, \frac{3}{2}\right)$.

Problem 37. A hyperbola passes through the point $P\left(\sqrt{2}, \sqrt{3}\right)$ and has foci at $(\pm 2, 0)$. Then the tangent to this hyperbola at P also passes through the point

1) $\left(2\sqrt{2}, 3\sqrt{3}\right)$
2) $\left(\sqrt{3}, \sqrt{2}\right)$
3) $\left(-\sqrt{2}, -\sqrt{3}\right)$
4) $\left(3\sqrt{2}, 2\sqrt{3}\right)$

Problem 38. If $5(\tan^2 x - \cos 2x) = 2\cos 2x + 9$, then find $\cos 4x$.

Problem 39. If $(x + \jmath y)^2 = 7 + 24\jmath$, then find $\left(7 + \sqrt{-576}\right)^{\frac{1}{2}} - \left(7 - \sqrt{-576}\right)^{\frac{1}{2}}$.

Problem 40. The sum of the first 15 terms of the series $3 + 7 + 14 + 24 + 37 + \ldots$ is $15k$. Find k.

Problem 41. Find

$$\lim_{x \to 0} \frac{\log(\sin 7x + \cos 7x)}{\sin 3x}. \tag{41.1}$$

Problem 42. Find the sum of the abscissae of the points where the curves $y = kx^2 + (5k + 3)x + 6k + 5, (k \in \mathbf{R})$, touch the x-axis.

Problem 43. The line $y = mx$ bisects the area of the region

$$\left\{(x, y) : 0 \leq x \leq \frac{3}{2}, 0 \leq y \leq 1 + 4x - x^2\right\}. \tag{43.1}$$

Find m.

Problem 44. Sketch the circle, which is the mirror image of the circle, $x^2 + y^2 - 2x = 0$, in the line, $y = 3 - x$.

Problem 45. Sketch the perpendiculars drawn from the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$ upon the tangent to it at the point $\left(\frac{3}{2}, \frac{5\sqrt{3}}{2}\right)$ and find their product.

Problem 46. Which one of the following points does not lie on the normal to the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ drawn at the point $\left(8, 3\sqrt{3}\right)$.

1) $\left(13, -\frac{1}{\sqrt{3}}\right)$
2) $\left(12, \frac{1}{\sqrt{3}}\right)$
3) $\left(11, \sqrt{3}\right)$
4) $\left(10, \sqrt{3}\right)$

Problem 47. Find the value of $\frac{1}{\cos 285^\circ} + \frac{1}{\sqrt{3}\sin 255^\circ}$.